

Non-Newtonian Dynamic Gravitational Field from The Longitudinally Asymmetric Rotating Objects

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Abstract

The dynamic shift of the center of mass for a rotating hemisphere prompts us the question of what might be its physical consequences. Despite the fact that accelerating object is known to create gravitational field, there is no known external dynamic gravitational force from a rotating sphere where the individual mass components are in constant acceleration. However, Thirring's 'induced centrifugal force' and the component of the force along the longitudinal axis inside a rotating spherical shell indicate that they are non-radiative dynamic forces which depend on ω^2 . In this report, Thirring's force is derived by considering the component-wise acceleration of the rotating hemisphere in the weak field approximation. This new analytic solution provides the gravitational explanation of the jet phenomena observed from the fast rotating cosmological bodies, which demands a major revision in our understanding of the universe since it suggests there exists a strong, long ranged, non-Newtonian dynamic gravitational force in our universe. This also raises an interesting question of how the strength of the dipole moment can be maximized for a given mass by configuring the specific geometrical shape of the rotating source.

The problem of the non-Newtonian gravitational force experienced by a test particle inside a rotating spherical shell has been considered by Thirring[1] in 1918. In his calculation within the weak field approximation, Thirring used the constant mass density ρ and the four velocity

$$\begin{aligned} u^1 &= \frac{\omega R \sin\theta \sin\phi}{\sqrt{1 - \omega^2 R^2 \sin^2\theta}} \\ u^2 &= \frac{\omega R \sin\theta \cos\phi}{\sqrt{1 - \omega^2 R^2 \sin^2\theta}} \\ u^3 &= 0 \\ u^0 &= \frac{1}{\sqrt{1 - \omega^2 R^2 \sin^2\theta}} \end{aligned} \quad (1)$$

and the length contraction

$$d^3x' = d^3x'' \sqrt{1 - \omega^2 R^2 \sin^2\theta} \quad (2)$$

for the rotating spherical mass shell to perform the integration in the rest frame of the source to evaluate Φ'_μ ,

$$\Phi'_\mu = 4 \int \frac{\rho u'_\mu u'^\nu d^3x'}{|\mathbf{r} - \mathbf{r}'|} \quad (3)$$

from which $h_{\mu\nu}$ can be calculated as

$$h_{\mu\nu} = \Phi_{\mu\nu} - \frac{1}{2}\Phi \quad (4)$$

In fact, one could have calculated the Φ'_μ in the rest frame of the observer by using the relativistic mass density in the same range of the radial integral and the resulting effects would have been the same since the integrand for Φ'_μ is the same for both cases. By such a method of calculation[1], Thirring has effectively circumvented the problem of the questionable rigidity of the spherical mass shell. On the other hand, physically, it is equivalent of taking the relativistic total mass-energy density $\gamma(\omega, \theta)\rho$ for the dynamic mass components of the shell and

then perform the integration in the observer's rest frame without concerning about the rigidity of the source.

For a test particle located close to the center of mass of the rotating spherical mass shell of radius R with the angular frequency ω , the Cartesian components of the acceleration (force/mass) have been shown to be given by, using the above method [1] [2] [3],

$$\begin{aligned}\ddot{x} &= \frac{M}{3R} \left(\frac{4}{5} \omega^2 x - 8\omega\nu_y \right) \\ \ddot{y} &= \frac{M}{3R} \left(\frac{4}{5} \omega^2 y + 8\omega\nu_x \right) \\ \ddot{z} &= -\frac{8M}{15R} \omega^2 z\end{aligned}\tag{5}$$

In regard to this problem, Bass and Pirani[4] have reported earlier that the 'induced centrifugal force' in the Thirring's geodesic equation[1][2][3] near the center of the rotating spherical mass shell arises as a consequence of the latitude dependent velocity distribution. From this observation, Cohen and Sarill[5] suggested that the 'induced centrifugal force' effect is due to the quadrupole moment. To investigate this problem analytically, using the longitudinal symmetry of the problem, one may write

$$\Phi_{\mu}^{\nu} = 4 \int \frac{\rho u'_{\mu} u'^{\nu} d^3 x'}{|\mathbf{r} - \mathbf{r}'|} = 8\pi \int_0^{\pi/2} \int \frac{\rho u'_{\mu} u'^{\nu} r'^2 \sin\theta dr' d\theta'}{|\mathbf{r} - \mathbf{r}'|} + 8\pi \int_{\pi/2}^{\pi} \int \frac{\rho u'_{\mu} u'^{\nu} r'^2 \sin\theta dr' d\theta'}{|\mathbf{r} - \mathbf{r}'|}\tag{6}$$

For $\omega R \ll 1$, the T^{00} component of the stress energy tensor becomes the dominant term in Eq. (6). By employing the Eq.s (1) and (2), Φ_{00} for the upper half of the sphere may be written by

$$\Phi_{0(half)}^0 = 8\pi \int_0^{\pi/2} \int \frac{\rho (\sqrt{1 - \omega^2 R^2 \sin^2 \theta''})^{-1} \sin\theta'' r''^2 dr'' d\theta''}{|\mathbf{r} - \mathbf{r}''|}\tag{7}$$

For the field outside the rotating hemispherical shell, the major contribution to the field from the half of the sphere in Eq. (7) is given by, up to the order $(1/r^2)$,

$$\phi = -\frac{M}{r} - \frac{d_z}{r^2} \cos\theta + O\left(\frac{1}{r^3}\right)\tag{8}$$

where

$$d_z = M\delta r_c = MR \left(\frac{1}{2} - \frac{\frac{1-\sqrt{1-\alpha}}{\alpha}}{\sqrt{\frac{1}{\alpha}} \sinh^{-1} \sqrt{\frac{\alpha}{1-\alpha}}} \right) \quad (9)$$

$$\alpha = \frac{\omega^2 R^2}{c^2} \quad (10)$$

for the hemispherical shell (flat side down) of radius R , total mass M , rotating with the angular frequency ω , where the origin of the coordinate system is located at the rest state ($\omega = 0$) center of mass of the hemisphere and δr_c is defined as the anomalous center of mass shift. The Eq.s (8) and (9) show that the strength of the dipole field depends on the magnitude of the anomalous shift of the center of mass which again depends on the geometrical configuration of the rotor. When the two dipole fields are superposed inside a spherical shell, it turns out that the resulting non-Newtonian ω^2 dependent gravitational force inside the spherical shell is formally identical to that of Thirring's 'induced centrifugal force'[1][2][3]. The major inverse r^3 forces from the two opposite dipole fields are canceled inside the sphere and there remain forces that behave like a harmonic oscillator in the z direction and that of a cascade in the radial direction. These remaining ω^2 dependent forces in the x, y, z directions close to the center of the sphere are given by

$$\begin{aligned} \ddot{x} &= \frac{2M}{R}\omega^2 x \\ \ddot{y} &= \frac{2M}{R}\omega^2 y \\ \ddot{z} &= -\frac{4M}{R}\omega^2 z \end{aligned} \quad (11)$$

Apart from the apparent formal resemblances, there are couple of discrepancies between this result and that of Thirring's. The first conspicuous one is the difference in the constant factor of $2/15$ between the two expressions. Also the information on the velocity dependent force is lost which is caused by the fact

that the other components of the stress-energy tensor have been ignored except T_{00} for the field outside of the source. The discrepancy in the constant factor would have been expected since the position $r=R/2$ is not far outside of the boundary of the source, while the $1/|r - r'|$ expansion for the dipole moment was made with the assumption $r > r' = R/2$. Therefore, the multipole potential for a rotating spherical shell may be written

$$\phi = -\frac{M}{r} + \frac{d_z/2}{|-(R/2)\hat{z} - \mathbf{r}|^2} \cos\theta' - \frac{d_z/2}{|(R/2)\hat{z} - \mathbf{r}|^2} \cos\theta'' + O\left(\frac{1}{r^3}\right) \quad (12)$$

where the angles θ' and θ'' are given by

$$\begin{aligned} \theta' &= \tan^{-1}\left(\frac{r \sin\theta}{r \cos\theta + R/2}\right) \\ \theta'' &= \tan^{-1}\left(\frac{r \sin\theta}{r \cos\theta - R/2}\right) \end{aligned} \quad (13)$$

respectively and d_z is given by the Eq. (9). The Eq. (12) is plotted in the three dimensional diagram of the potential in Fig. 1, which shows the quadrupole feature inside the rotating spherical mass shell as proposed earlier by Cohen and Sarill [5].

The problem is very similar to that of the electromagnetic vector potential from a circular ring of radius a with current I . It is well known that the azimuthal component (the only non-zero term due to the symmetry) of the vector potential for both inside and outside of the radius a of the ring is approximately given by

$$A_\phi(r, \theta) = \frac{\pi I a^2 r \sin\theta}{c(a^2 + r^2)^{3/2}} \left(1 + \frac{15a^2 r^2 \sin^2\theta}{8(a^2 + r^2)^2} + \dots \right) \quad (14)$$

For $r \gg a$, the leading term of this potential depends on $1/r^2$ indicating the dipole effect. It also gives the details of the potential inside the radius a without singularity. Following this example, one may introduce a weight parameter η into the gravitational dipole potential

$$\phi_{dipole} \propto \frac{-r}{(\eta^2 + r^2)^{3/2}} \quad (15)$$

so that the potential behaves without singularity for $r < r'$, where η represents the parametrized radius of the physical object. For a non-spherical body like a hemisphere, for example, one may assign the parameter tentatively a virtual physical dimension of a shell

$$\eta = \sqrt{0.1}R \quad (16)$$

which is about one third of the radius R of the sphere. In this case, the corrected non-Newtonian force near the center of the sphere is reduced approximately by a factor $1/8$ from the one in Eq. (11). This is close to the value $2/15$ which gives exactly Thirring's induced centrifugal force. The above discussions suggest that the w^2 dependent forces in Thirring's result are mainly from the partially canceled dipole effect which arise due to the subtractive contribution to F_z and the additive ones for F_x, F_y from the two dipole moments respectively. In regard to this problem, Bass and Pirani[4] also have shown that the centrifugal force term arises as a consequence of the latitude dependent velocity distribution which generates an axially symmetric (non-spherical) mass distribution, which casts doubts on the centrifugal force interpretation of the Thirring's result since the rotating cylindrical object would not have such latitude dependent density distribution and there will be no corresponding centrifugal force for the cylindrical object, contrary to our expectation. These difficulties remain even when the contribution from elastic stress is included, which led Bass and Pirani to conclude that there was an apparent conflict with Mach's principle. Following this observation, Cohen and Sarill reported that the centrifugal term from Thirring's solution for a rotating spherical mass actually represents a quadrupole effect[5] by a deductive argument and suggested an alternative solution[6] (also previously by Pietronero[7]) for the centrifugal force in general relativity using the flat space metric in rotating coordinates. By employing the result in Eq. (15), the potential for a rotating spherical mass shell for both inside and out may be written, up to the dipole moment

$$\phi = V(r) + \frac{|-(R/2)\hat{z} - \mathbf{r}|d_z/2}{(\eta^2 + (-(R/2)\hat{z} - \mathbf{r})^2)^{3/2}} \cos\theta' - \frac{|(R/2)\hat{z} - \mathbf{r}|d_z/2}{(\eta^2 + ((R/2)\hat{z} - \mathbf{r})^2)^{3/2}} \cos\theta'' + O\left(\frac{1}{r^3}\right) \quad (17)$$

where

$$\begin{aligned}
V(r) &= -M/r && \text{for } r > R \\
&= -M/R && \text{for } r \leq R
\end{aligned}
\tag{18}$$

and the angles θ' and θ'' are given by Eq. (13).

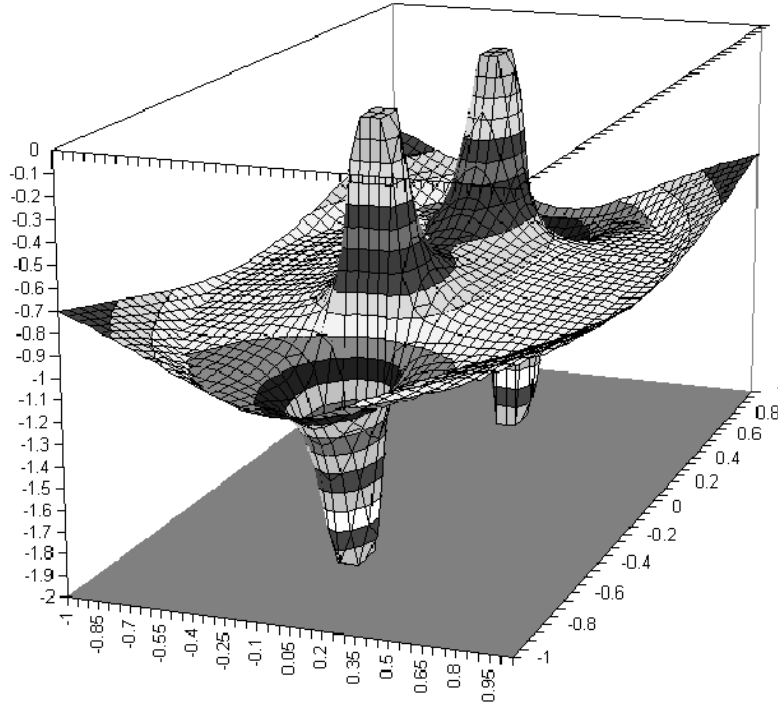


Fig. 1. A close up view of the computer generated 3-D diagram for the multipole gravitational potential inside a spherical mass shell rotating along the z axis. All the constants are set equal to 1 ($M = G = R = c = 1$) and the value of the parameter η is set equal to zero. The anomalous center of mass shift δr_c for the potential in the diagram is $0.05R$ which corresponds to the case $\omega R = 0.045c$. The variable range of δr_c is from 0 to $0.5R$.

This potential is plotted in Fig. 1 for a close up view of the inside field of the sphere for $\omega R = 0.05c$. The actual potential inside the rotating sphere must include the higher order terms which are similar to the ones in Eq. (14) to correctly represent the inner potential. In general, the behavior of the potential in the longitudinal axis suggests (Fig. 2) the possibility that the particles traveling into the attractive dipole potential well along the z axis will be repelled back to where they have come from depending on the rotational frequencies

that support the height of the peaks. This repulsive potential peak determines the range of the linear orbital distances that the particles may travel back and forth from the poles to the far outsides along the z axis.

Since there is no compelling evidence that the plasma and magnetic field must be generated inside rotating ultra-compact bodies, where the electronic orbital states have been long before collapsed, one may suspect that the superposed dipole effect may have been the major driving force behind the jet phenomena in some of the fast rotating cosmological objects.

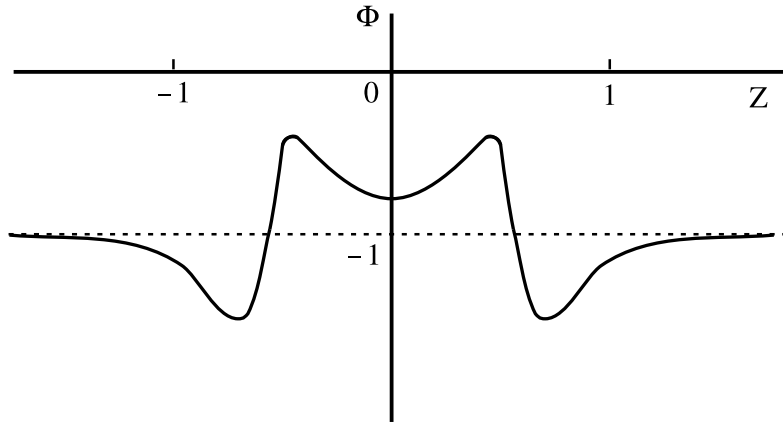


Fig. 2. An example of the superposed dipole field (Eq. 17) along the longitudinal (z) axis ($\eta^2 = 0.02$, $M=G=c=R=1$, $\omega R = 0.05c$)

Collisions and vibrations of particles coming in and going out following the linear trajectories may generate wide range of frequencies of radiations observed from quasars. This view point is supported by the fact that the dipole field is long ranged and the strongest next to that of the monopole. The long range potential dip around the equator in the diagram also indicates that there exists a tendency of the cluster formation around the equatorial plane of rotating celestial bodies. Since the dipole field depends on the geometrical shape of the rotor, it is interesting to find out what will be the most efficient geometrical configuration a rotor could take for given mass to produce the maximum dipole moment. This question is interesting since the object under the influence of multipole gravitational potential will tend to take a shape which is different

from the spherical symmetry. The strength of the dipole moment defined by $M\delta r_c$ may be treated as an independent gravitational charge that determines the gravitational dipole field around the object the same way as one determines the electric or magnetic dipole moment. Since this quantity depends critically on the longitudinal asymmetry of the rotor, one may start by assuming that the contour of the rotor assumes the form $z = a|x|^b$, where z is the rotation axis.

To be able to determine the relative efficiency of the dipole rotor, a constraint must be imposed on the total rest mass of the rotor. Thus, the problem reduces into mathematically determining the geometrical contour that gives the maximum shift of the center of mass for a given mass of the rotor. The total rest mass of the solid object, the contour of which is expressed by the equation $z = a|x|^b$ (for $-x_s < x < x_s$, $a > 0$, $b > 0$), is written in the rotor's rest frame by

$$M_0 = \rho V = \rho \int \pi abx^{b+1} dx = \rho \frac{\pi ab}{b+2} x_s^{b+2} \quad (19)$$

where x_s is the radius of the widest area of the object in the plane of rotation and ρ is the density of the material. The z component of the dipole moment of the object rotating with the angular frequency ω along the z axis is given by, in the observer's frame, following the method of Thirring's for $r > r'$ (note that x , y components are zero due to the symmetry),

$$Mz = \int \rho \pi abx^{b+1} ax^b \frac{dx}{\sqrt{1 - \frac{\omega^2 x^2}{c^2}}} \quad (20)$$

Since we are interested in the case for $\omega x \ll c$, the integral can be approximated to be

$$Mz = \int \rho \pi a^2 bx^{2b+1} \left(1 + \frac{\omega^2 x^2}{2c^2} \right) dx \quad (21)$$

which gives the result

$$Mz \approx \rho \pi a^2 b \left(\frac{1}{2b+2} x_s^{2b+2} + \frac{\omega^2}{2c^2} \frac{1}{2b+4} x_s^{2b+4} \right) \quad (22)$$

where the first term is the fictitious dipole moment of the rotor. This term can

be eliminated by the coordinate translation. The second term is due to the latitude dependent velocity distribution which causes the coordinate independent, anomalous shift of the center of mass that may be written, using the Eq. (19), as

$$d_z = M\delta_z \approx \frac{\omega^2 M_0^2}{4\pi\rho c^2} \left(1 + \frac{2}{b}\right) \quad (23)$$

For instance, for a rotating cylindrical mass shell, the ωx in Eq. (20) would have to be replaced by ωx_0 where x_0 is the radius of the cylinder. Consequently, the γ factor in Eq. (20) becomes a constant and the corresponding expression for Eq. (22) would not have the second term which represents the gravitational dipole moment. This is in accordance with the observation reported by Bass and Pirani [4] that Thirring's 'induced centrifugal force' is actually due to the latitude dependent velocity distribution which does not exist in a rotating cylindrical mass. The above result indicates that the smaller the value of b , the greater the efficiency of the dipole effect. However, the value of b affects the other constraint since the total rest mass of the rotor depends both on the values of a and b respectively by the relation (19).

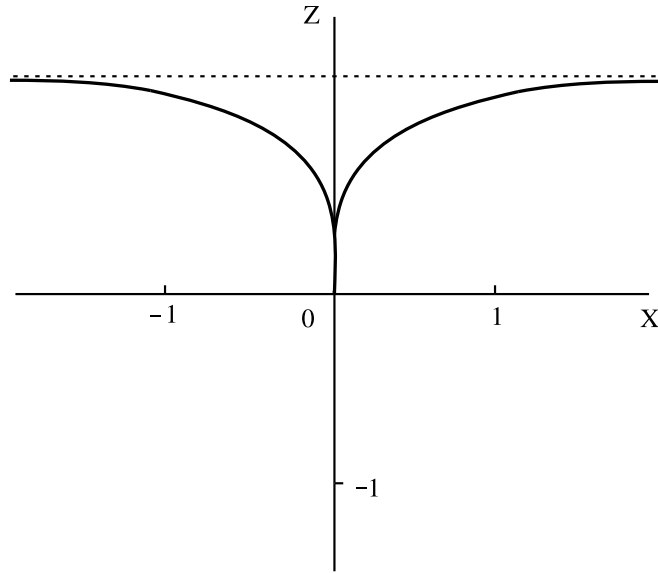


Fig. 3. The shape of an efficient dipole rotor which has the contour equation $z = |x|^{1/5}$

By assuming that b is much smaller than 1, the expression (19) can be rewritten

$$M_0 \approx \rho \frac{\pi ab}{2} x_s^2 \quad (24)$$

The smaller b requires correspondingly the larger a to maintain the constant rest mass. The Eq. (23) also suggests that the lower the density of the material comprising the rotor, the greater the effect of the dipole moment when the constraint is on the mass of the rotor, in which case a larger x_s would be required. In general, the form of the curve for small b ($b < 1$) suggests that the outer shape of the rotor must be like a cusped funnel, the contour of which is plotted in Fig. 3, in the first order approximation to have the efficient dipole moment effect. For extremely small b , the side view of the contour becomes the shape of a letter T, in which case the efficiency may approach the maximum. An interesting observation in regard to this problem is that a fast rotating galaxies tend to have the shape of two superposed opposite funnels (Fig. 4).

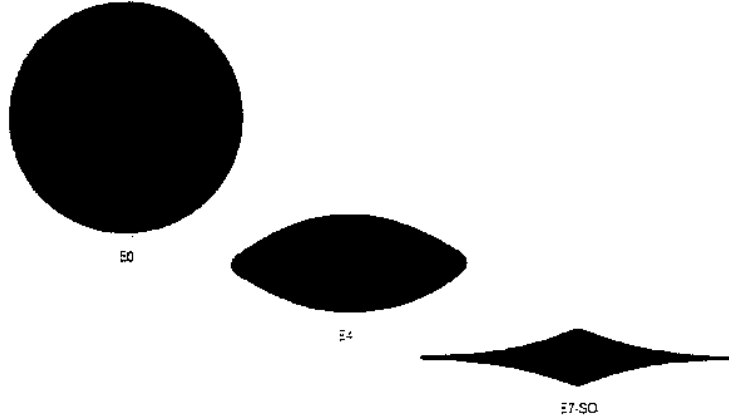


Fig. 4. The shape of the galaxies in the order of increasing rotational frequencies.

It seems that the cosmological objects find the most efficient contour by itself, depending on the rotational frequencies of the whole body. In the first order, one observes the celestial bodies take the form of a spherical shape due to the isotropy of the monopole field and in the second order, when the fast rotational motion is involved, the cosmological bodies transform into a longitudinal axially symmetric shape like that of a superposed funnel or a disc with long jets. As

can be seen from Fig. 2, mass tends to be accumulated near the poles which would transform the spherical mass into a cusped shape which would again help increase the efficiency of the dipole moment. This process is self perpetuating and suggests a strong possibility that the jets in the black hole accretion discs are an extended form of the general configuration of the fast rotating cosmological bodies resulting from the dynamic gravitational dipole field.

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