An Isolated Gravitational Dipole Moment Placed at The Center of the Two Mass Pole Model Universe

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Abstract

The unexpected dynamic shift of the center of mass for a rotating hemisphere is shown to produce the general relativistic dipole field in the macroscopic scale. This prompts us the question of what might be its cosmological implications. The uniformly rotating sphere has the effect of the latitude dependent mass density distribution as reported by Bass and Pirani which is the cause of the 'induced centrifugal force' in the Thirring's geodesic equation near the center of the rotating spherical mass shell. On the other hand, one would expect the constant acceleration of the mass components may cause a general relativistic gravitational field. The component-wise accumulation of this effect has been shown to appear as the non zero gravitational dipole moment in a rotating hemispherical mass shell. The present report discusses this non-Newtonian force experienced by a gravitational dipole moment placed at the center of the two mass pole model universe and its relevance to the observed anomalous red shift from far away galaxies.

In 1911, von Laue [1] presented an argument of a general nature against the existence of a rigid body in relativity theory. It is based on the fact that a rigid body is expected to have a finite number of degrees of freedom, while, on the other hand, one can set up N disturbances near N separated points of the body, and they will be non-interacting, ie., independent for a sufficiently short interval of time (because of the finite propagation time required by relativity theory) so that there are at least N degrees of freedom, where N can be increased indefinitely for a continuous medium. Other difficulties include the apparent paradox pointed out by Ehrenfest [2]. Since the elements of the circumference of a circle in a rotating disk move in the direction of their instantaneous velocities, one would anticipate a diminished value for the circumference. However, since the elements of the radii of the circle everywhere move normally to the direction of their velocities, no such contraction for the length of the radius would be expected. This is the standard argument from which it is concluded that the 'geometry' of the rotating body can not be Euclidean [3]. On the other hand, Hill[4] pointed out (as von Laue [5] had done earlier) that a rotating body would have a limit to the radius it could have, for the velocity would vary linearly with the radius and would exceed that of light for $r > 1/\omega$ and that the speed distance law must be nonlinear while the Euclidean geometry is maintained. However, Rosen [6] argued using a covariant formulation that the speed distance law ($v = r\omega$) must be preserved and that the spatial geometry on the surface of a rotating disk is non-Euclidean. For these reasons, as pointed out by Phipps [7], the rigidity of a metric standard or of any extended structure such as a disk has never found a consistent [8] relativistic definition as a purely kinematic attribute. Rigidity has therefore generally been conceived [9] as a nonexistent physical property. However, the experiment suggested by Weinstein [10] and later performed by Phipps [7] exhibited a 'rigid body' (to order v^2/c^2) and thus provides evidence that rigidity is not always a physically impermissible idealization. He showed that straight lines on the disk surface remain straight on hyperplanes of constant laboratory time is consistent with both the classical and the Born[11] definition of rigidity. The problem of the non-Newtonian gravitational force experienced by a test particle inside a rotating spherical shell has been considered by Thirring [12] in 1918. In his calculation within the weak field approximation, Thirring used the constant mass density r and the four velocity

$$
\begin{aligned}
u^1 &= \frac{\omega R sin\theta sin\phi}{\sqrt{1-\omega^2 R^2 sin^2\theta}} \\
u^2 &= \frac{\omega R sin\theta cos\phi}{\sqrt{1-\omega^2 R^2 sin^2\theta}} \\
u^3 &= 0 \\
u^0 &= \frac{1}{\sqrt{1-\omega^2 R^2 sin^2\theta}}
\end{aligned}
\tag{1}
$$

and the length contraction

$$
d^3x' = d^3x''\sqrt{1-\omega^2 R^2 sin^2\theta} \tag{2}
$$

for the rotating spherical mass shell to perform the integration in the rest frame of the source to evaluate Φ_μ^ν,

$$
\Phi_\mu^\nu = 4\int \frac{\rho u'_\mu u'^\nu d^3x'}{|\mathbf{r}-\mathbf{r}'|} \tag{3}
$$

from which $h_{\mu\nu}$ can be calculated as

$$
h_{\mu\nu} = \Phi_{\mu\nu} - \frac{1}{2}\Phi \tag{4}
$$

In fact, one could have calculated the Φ_μ^ν in the rest frame of the observer by using the relativistic mass density in the same range of the radial integral and the resulting effects would have been the same since the integrand for Φ_μ^ν is the same for both cases. By such a method of calculation[12], Thirring has effectively circumvented the problem of the questionable rigidity of the spherical mass shell. On the other hand, physically, it is equivalent of taking the relativistic total mass-energy density $\gamma(\omega,\theta)\rho$ for the dynamic mass components of the shell and then perform the integration in the observer's rest frame without concerning about the rigidity of the source.

For a test particle located close to the center of mass of the rotating spherical mass shell of radius R with the angular frequency ω, the Cartesian components of the acceleration (force/mass) have been shown to be given by, using the above method [13] [14] [15],

$$\begin{aligned}\ddot{x} &= \frac{M}{3R}(\frac{4}{5}\omega^2 x - 8\omega\nu_y) \\ \ddot{y} &= \frac{M}{3R}(\frac{4}{5}\omega^2 y + 8\omega\nu_x) \\ \ddot{z} &= -\frac{8M}{15R}\omega^2 z \end{aligned} \qquad (5)$$

where v_y and v_x are the y and x components of the velocity of the test particle, respectively. The test object placed slightly off the center of mass toward the z axis will be subjected to the harmonic oscillation according to the third formula given above. The motion of the particle in the x, y plane is spiraling away from the rotation axis with the force proportional to $\omega^2 r$ where r is the radial distance from the symmetry axis in the x, y plane, which has been interpreted as the indication of the existence of the induced centrifugal force in general relativity in accordance with the Mach's principle.

On the other hand, the field outside of the rotating spherical mass shell has not been scrutinized in the same fashion as Thirring did in his work on the interior solution. Part of the conceptual difficulty was that it has been worked out in the general formalism in the multipole expansion of the linearized theory for $r > r'$ in which the Newtonian potential has been derived and the conclusion has been reached that the rotating spherical mass does not have the dipolar effect [16] while Thirring's work on the problem inside the rotating shell shows interesting longitudinal linear force effect in the z direction which is characteristically dipolar.

To elucidate the problem, it is necessary to reexamine the dipole term in the weak field approximation for the rotating hemispherical mass shell. By following the method of Thirring [12] outside of the source ($r > r'$), assuming also that the mechanical stress in the shell is small, it can be shown that the dipole field calculated from the rotating hemispherical shell has the non-zero value which cannot be eliminated by the coordinate translation. By keeping only the T^{00}, the corresponding gravitational dipole moment is given by, in exact form

$$d_z = M\delta r_c = MR\left(\frac{1}{2} - \frac{\frac{1-\sqrt{1-\alpha}}{\alpha}}{\sqrt{\frac{1}{\alpha}}\sinh^{-1}\sqrt{\frac{\alpha}{1-\alpha}}}\right) \qquad (6)$$

$$\alpha = \frac{\omega^2 R^2}{c^2} \tag{7}$$

for a bowl shaped hemispherical shell of radius R and mass M placed on the x, y plane, where d_z is defined as the anomalous shift of the center of mass which doesn't depend on the choice of the specific coordinate system. For $\omega R << c$, the d_z can be approximated to be

$$\delta r_c = \frac{\omega^2 R^3}{24c^2} \tag{8}$$

As the result of this non zero gravitational dipole moment, the field outside of the rotating hemispherical mass shell of radius R is given by, up to the approximation,

$$\phi = -\frac{M}{r} - \frac{d_z}{r^2} cos\theta + O(\frac{1}{r^3}) \tag{9}$$

where M is the total mass of the source, q is measured with respect to the positive z axis and d_z is given by Eq. (6). This dipole field has the force line which is exactly the same as that of the electric or magnetic dipole moment and diverges at $r = 0$ since the expansion for $1/|r - r'|$ has been made with the assumption $r > r'$. The Cartesian components of the dipole force can be written by

$$\begin{aligned} F_{xdipole} &= -\frac{\partial}{\partial x}(\frac{Gd_z}{r^2}\cos\theta) = Gd_z\frac{3zx}{r^5} \\ F_{ydipole} &= -\frac{\partial}{\partial y}(\frac{Gd_z}{r^2}\cos\theta) = Gd_z\frac{3zy}{r^5} \\ F_{zdipole} &= -\frac{\partial}{\partial z}(\frac{Gd_z}{r^2}\cos\theta) = Gd_z\frac{-r^3 + 3z^2 r}{r^6} \end{aligned} \tag{10}$$

where the center of the coordinate system is located at the distance $R/2$ from the center of the sphere. The gravitational dipole moment points to the negative z axis (opposite to the direction of the center of mass shift) when the hemisphere is placed like a bowl on the x, y plane, independent of its direction of the angular velocity. The direction of the z component of the force along the symmetry axis in Eq. (10) is uniformly toward the positive z axis except between the singular points. Thus, a test particle in front of the domed side of the hemispherical shell would be attracted toward inside and the one near the flat side will be repelled.

Using this result, one may attempt to calculate the force experienced by a test particle at the position (x, y, z), where $|x| = |y| = |z| << R$, by attaching the two rotating hemispherical shells to form a sphere and adding the two opposite dipole forces near the center. The center of the sphere is now at the boundary of the hemisphere, separated uniformly from the shell by the distance R. Therefore, since the point of interest is not exactly on the singular point of the volume integral of the potential for the hemispherical shell, one would expect the field near the center of the sphere will behave reasonably well within certain amount of expected error.

Since the dipole field is expressed in the coordinate system in which the origin is located at the center of mass of the hemisphere ($R/2$ from the center of the sphere), the position of the test particle becomes $(x, y, R/2 - z)$ with respect to the coordinate system the origin of which is located at the center of mass of the upper hemisphere and $(x, y, R/2 + z)$ with respect to that of the lower one. Using the above equations and the relation (8) for the expression for d_z, one obtains

$$\begin{aligned} \ddot{x} &= \frac{2M}{R}\omega^2 x \\ \ddot{y} &= \frac{2M}{R}\omega^2 y \\ \ddot{z} &= -\frac{4M}{R}\omega^2 z \end{aligned} \tag{11}$$

for the force experienced by the test particle near the center of the rotating spherical mass shell of mass M for $\omega R << c$. Apart from the apparent formal resemblances, there are couple of discrepancies between this result and that of Thirrings. The first conspicuous one is the difference in the constant factor of $2/15$ between the two expressions. Also the information on the velocity dependent force is lost which is caused by the fact that the other components of the stress-energy tensor have been ignored except T^{00} for the field outside of the source. The discrepancy in the constant factor would have been expected since the position $r = R/2$ is not far outside of the boundary of the source, while the $1/|r - r'|$ expansion for the dipole moment was made with the assumption $r > r' = R/2$. This problem is very similar to that of electromagnetic vector potential from a circular ring of radius a with current I. It is well known that the azimuthal component (the only non-zero term due to the symmetry) of the vector potential for both inside and outside of the radius a of the ring is approximately given by

$$A_\phi(r,\theta) = \frac{\pi I a^2 r sin\theta}{c(a^2+r^2)^{3/2}} \left(1 + \frac{15a^2r^2sin^2\theta}{8(a^2+r^2)^2} + ...\right) \tag{12}$$

For $r >> a$, the leading term of this potential depends on $1/r^2$ indicating the dipole effect. It also gives the details of the potential inside the radius a without singularity. Following this example, one may introduce a weight parameter η into the gravitational dipole potential

$$\phi_{dipole} \propto \frac{-r}{(\eta^2+r^2)^{3/2}} \tag{13}$$

so that the potential behaves without singularity for $r < r'$, where η represents the parametrized radius of the physical object. For a non-spherical body like a hemisphere, for example, one may assign the parameter tentatively a virtual physical dimension of a shell

$$\eta = \sqrt{0.1}R \tag{14}$$

which is about one third of the radius R of the sphere. In this case, the corrected non-Newtonian force near the center of the sphere is reduced approximately by a factor 1/8 from the one in Eq. (11). This is close to the value 2/15 which gives exactly Thirring's induced centrifugal force. The above discussions suggest that the ω^2 dependent forces in Thirring's result are mainly from the partially canceled dipole effect which arise due to the subtractive contribution to F_z and the additive ones for F_x, F_y from the two dipole moments respectively.

In regard to this problem, Bass and Pirani [17] also have shown that the centrifugal force term arises as a consequence of the latitude dependent velocity distribution which generates an axially symmetric (non-spherical) mass distribution, which casts doubts on the centrifugal force interpretation of the Thirring's result since the rotating cylindrical object would not have such latitude dependent density distribution and there will be no corresponding centrifugal force for the cylindrical object, contrary to our expectation. These difficulties remain even when the contribution from elastic stress is included, which led Bass and Pirani to conclude that there was an apparent conflict with Mach's principle. Following this observation, Cohen and Sarill reported that the centrifugal term from Thirring's solution for a rotating spherical mass actually represents a quadrupole effect[18] by a deductive argument and suggested an alternative solution [19] (also previously by Pietronero[20]) for the centrifugal force in general relativity using the flat space metric in rotating coordinates.

The discussion so far indicates that the superposed dipole field description gives the quadrupole effect as proposed by Cohen and Sarill which is identical to that of Thirring's 'induced centrifugal force' near the center of the sphere. This also provides a detailed look at the general field configuration outside the rotating spherical mass shell, up to the component T^{00}, which is obtained by adding the fields created by the two opposite dipole moments separated by the distance R in addition to the monopole field,

$$\phi = -\frac{M}{r} + \frac{d_z/2}{|-(R/2)\hat{z} - \mathbf{r}|^2} cos\theta' - \frac{d_z/2}{|(R/2)\hat{z} - \mathbf{r}|^{\mathbf{2}}} cos\theta'' + O(\frac{1}{r^3}) \tag{15}$$

where the angles θ' and θ'' are given by

$$\begin{aligned} \theta' &= tan^{-1}\left(\frac{rsin\theta}{rcos\theta + R/2}\right) \\ \theta'' &= tan^{-1}\left(\frac{rsin\theta}{rcos\theta - R/2}\right) \end{aligned} \tag{16}$$

respectively and d_z is given by the Eq. (6). By employing the result in Eq. (13), one may write the potential for a rotating spherical mass shell for both inside and out,

$$\phi = V(r) + \frac{|-(R/2)\hat{z} - \mathbf{r}| d_z/2}{(\eta^2 + (-(R/2)\hat{z} - \mathbf{r})^2)^{3/2}} cos\theta' - \frac{|(R/2)\hat{z} - \mathbf{r}| d_z/2}{(\eta^2 + ((R/2)\hat{z} - \mathbf{r})^2)^{3/2}} cos\theta'' + O(\frac{1}{r^3}) \tag{17}$$

where

$$\begin{aligned} V(r) &= -M/r \qquad \text{for } r > R \\ &= -M/R \qquad \text{for } r \leq R \end{aligned} \tag{18}$$

and the angles θ' and θ'' are given by Eq. (16).

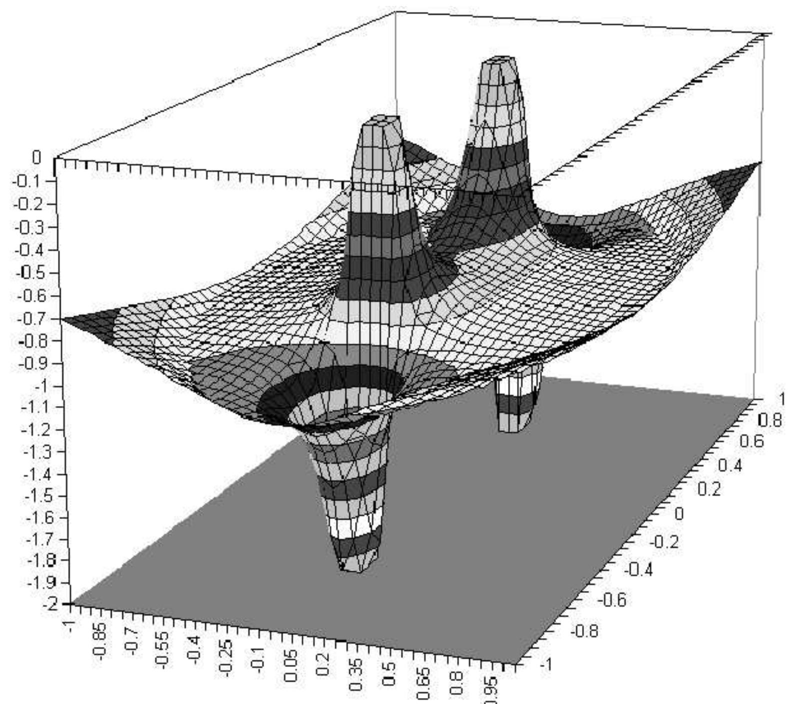

Fig. 1. A close up view of the computer simulated 3-D diagram for the multipole gravitational potential inside a spherical mass shell rotating along the z axis. All the constants are set equal to 1 ($M = G = R = c = 1$) and the value of the parameter η is set equal to zero. The anomalous center of mass shift δr_c for the potential in the diagram is 0.05R which corresponds to the case $\omega R = 0.045c$. The maximum variable range of δr_c is from 0 to 0.5R.

The potential diagram in Figure shows four distinctive poles indicating that it is indeed a quadrupole moment created by superposing two opposite dipoles. One of the major consequences of this potential is that there exist two strong repulsive potential peaks close to the center along the z axis, which is unusual for the gravitational force. Since it is generally believed that the repulsive gravitational potential can not be generated by ordinary matter, this is a puzzling event since there exists a pair of virtual negative mass poles as well as the positive ones inside a rotating spherical mass shell. The saddle point created by this two positive peak potentials near the center is the cause of Thirrings 'centrifugal force' as discussed above. This quadrupole feature disappears as soon as the rotational frequency becomes zero as can be seen from Eq. (17).

The behavior of the potential in the longitudinal axis also indicates the possibility that the particles traveling into the attractive dipole potential well along the z axis will be repelled back to where they have come from depending on the rotational frequencies that support the height of the peaks. This repulsive potential peak determines the range of the linear orbital distances that the particles may travel back and forth from the poles to the far outsides along the z axis.

Since there is no compelling evidence that the plasma and magnetic field must be generated inside rotating ultra-compact bodies, where the electronic orbital states have been long before collapsed, one may suspect that the superposed dipole effect may have been the major driving force behind the jet phenomenon in some of the fast rotating cosmological objects. This view point is supported by the fact that the dipole field is long ranged and the strongest next to that of the monopole. The long range potential dip around the equator in the diagram also indicates that there exists a tendency of the cluster formation around the equatorial plane of rotating celestial bodies.

The information that has been lost by neglecting the component T_{ik} includes the one from the elastic stress which has been considered by Bass and Pirani [17] as discussed above and also the velocity dependent (Coriolis) force in Eq. (5) which depends linearly on the rotational frequency ω. Contributions from these terms may be added into the potential without the loss of generality of the strong dipole feature. On the other side of the problem, the effect of the 'centrifugal force' from rotating spherical objects has been investigated by many researchers and a result has been recently reported by Gupta et al. [21]. As r increases in the equatorial plane, the saddle potential progresses toward the plateau which is a constant potential region. As can be seen from Figure, this transition is smooth without abrupt change of curvature. The force reaches the peak at a certain radius r_i from the center where the slope of the potential is the steepest and then gradually becomes zero as the potential reaches the plateau. This prediction of the general behavior of the maximum of the 'centrifugal force' from the potential (16) and (17) is consistent with the result (Fig.1) of Gupta et al. [21]. The reversal (sign change) effect is not present in this case since the object is made of a hollow shell. Although the main cause of the above result may seem like due to the mass increase from the latitude dependent velocity distribution of a rotating spherical mass as reported by Bass and Pirani [17], the corresponding permanent increase of the mass density would not produce the dipole effect since there would not be the dynamic shift of the center of mass in such case. Therefore, it is clear that this dynamic effect is due to the collective contribution from the uniform acceleration of the mass components in the rotating sphere.

The above discussion was based on the fact that the analytic interior solution of a rotating spherical mass shell exhibits a multipole potential which suggests there exist physically meaningful gravitational dipole moment in our universe contrary to the general belief that it doesn't. This argument is supported by the observation that the linear orbitals of the particles along the two poles of a rotating spherical mass resembles closely the observed jets from the fast rotating cosmological bodies. On the other hand, it becomes immediately obvious that a rotating asymmetrical body can have isolated gravitational dipole moment contrary to the generally known interpretation of the linearized theory. An independent dipole moment posseses all the dynamics that an isolated, controllable magnet would behave in the magnetic monopole universe.

In relation to this problem, one of the widely known mysteries of our current cosmology is the presence of the anomalous red shift observed from some of the far away galaxies. This has been a serious problem in cosmology since the current model of the universe does not allow such mode of movement of a galaxy assuming that the relativistic Doppler shift correctly represents the relative velocity between the observer and the observed. The theory of the big bang associated with Hubbles expansion law prohibits motions of the galaxies other than the uniform separation between any two of the galaxies. The conceptual model of an isolated gravitational dipole moment placed in the matter filled universe actually solves the problem of the anomalous red shift by simply assuming that the specific galaxy possesses the non zero net gravitational dipole moment in the direction of our galaxy. If we assume that at the time of the big bang some of the chunk of the matter came off with an asymmetric shape of the body with non zero rotational frequency, they will eventually acquire velocities unrelated to the uniform Hubble expansion. Even if the detached body may change its shape in time to become a longitudinally symmetric structure, the accumulated linear momentum will remain to be observed as an anomalous red shift in the spectrum.

Assuming that the universe can be modeled to consist of two large mass poles in the front and back of the dipole separated by the distance r with mass M/2 where M is the mass of the universe, one can calculate the net directional force on the dipole moment to be

$$F = \frac{2dM}{(r/2)^3} \tag{19}$$

where M is the mass of the universe and r the distance between the two mass poles. The direction of the force is toward the positive (pointed) side of dipole. The influence from the monopole force is canceled because the dipole moment is placed exactly in the middle of the two mass poles of the model universe. The next question is if the observed galaxies which exhibit anomalous red shift indeed have asymmetrical shape with respect to its longitudinal axis. It is something that may need to be verified by observational astronomy if indeed they do have asymmetrical configuration along the rotation axis. From the above considerations, it is obvious that we have a much more flexible model of the universe by including the dipole force in our system. The nature of this force is that, while it has been formally predicted by general relativity, it has not been fully recognized by the scientific community because the traditional treatment of the weak field approximation for a spherically symmetric body has concluded that there is no dipole term. However, since Thirrings solution for the 'induced centrifugal force' turned out to represent the partially canceled dipole field inside a rotating spherical mass shell, it no longer justifies to neglect the gravitational dipole moment.

The major consequences of this solution is that an axisymmetric yet longitudinally asymmetric rotating object can have the isolated dipole gravitational moment which is virtually identical in dynamics to the magnetic dipole moment placed in a sea of the uniform magnetic monopoles. By this result, two of the main cosmological mysteries of our time have become trivial consequences of the gravitational dipole moment that is imbedded in general relativity.

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